Longitudinal and Transverse Form Factors from ^{12}C *

F. A. Majeed[1,†]

[1] *Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, 21941-972 Rio de Janeiro, RJ, Brazil*

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Electron scattering form factors from ^{12}C have been studied in the framework of the particle-hole shell model. Higher configurations are taken into account by allowing particle-hole excitations from the $1s$ and $1p$ shells core orbits up to the $1f$-$2p$ shell. The inclusion of the higher configurations modifies the form factors markedly and describes the experimental data very well in all momentum transfer regions.

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I. INTRODUCTION

Shell model calculations, carried out within a model space in which the nucleons are restricted to occupy a few orbits are unable to reproduce the measured static moments or transition strengths without scaling factors. Inadequacies in the shell model wavefunctions are revealed by the need to scale the matrix elements of the one-body operators by effective charges to match the experimental data. However, the introduction of effective charges may bring the calculated transition strengths which are defined at the photon point, as well as, the form factors at the first maximum, closer to the measured values, but the non-zero momentum transfer (q) values might deviate appreciably from the measured values [1].

Electron scattering at 200 MeV on ^{12}C and ^{13}C, have been studied by Sato *et al.* [2]. The effect of higher configurations wavefunctions are included in the work of Bennhold *et al.* [3]. Booten *et al.* [4] investigated the higher configurations contributions on some p-shell nuclei. Coulomb form factors of C2 transitions in several selected p-shell nuclei are discussed by Radhi *et al.* [5] taking into account core-polarization effects. Configuration mixing shell model has been recently used [6] to study the isovector states of ^{12}C in the framework of particle-hole theory. The calculations are quite successful and describe the experimental form factors very well for all momentum transfer regions.

The purpose of the present work is to include higher-energy configurations by allowing excitation from $1s$ and $1p$ shells core orbits up to the $1f$-$2p$ shell. The configurations which include the higher configurations is called the extended space configurations. The ground state of ^{12}C is taken to have closed $1s_{1/2}$ and $1p_{1/2}$ shells. The states expected to be most strongly excited from closed-shell nuclei are linearly combination of a configurations in which one nucleon has been raised to a higher shell, forming pure single-particle-hole state [7]. This approximation is called Tamm-Dancoff approximation (TDA)[8]. The dominant dipole, quadrupole and multipole T=1 single particle-hole states of ^{12}C are considered with the framework of the harmonic oscillator (HO) shell model. The Hamiltonian is diagnoalized in the space of the single-particle hole states, in the presence of the modified surface delta interaction (MSDI) [9]. The space of the single-particle-hole states include all shells up to $2p_{1/2}$ shell. Admixture of higher configurations is also considered. A comparison of the calculated form factors using this model with the available experimental data for the dominantly T=1 states are discussed.

II. THEORY

The ground state of ^{12}C is taken to have closed $1s_{1/2}$ and $1p_{3/2}$ shells, and is represented by Ψ_0. The particle-hole state formed by promoting one particle from the shell-model ground state. The particle-hole state of the total Hamiltonian is represented by $\Phi_{JM}(ab^{-1})$ with labels (a) for particles with quantum numbers $(n_a\ell_a j_a)$ and (b) for holes with quantum numbers $(n_b\ell_b j_b)$. The state $\Phi_{JM}(ab^{-1})$ indicating that a particle was vacated from j_b and promoted to j_a.

The excited state wavefunction can be constructed as a linear combinations of pure basis $\Phi^{,s}$ as [7]

$$\Psi^n_{JM} = \sum_{ab} \chi^J_{ab^{-1}} \Phi_{JM}(ab^{-1}), \tag{1}$$

where the amplitude $\chi^J_{ab^{-1}}$ can be determined from a diagonalization of the residual interaction. By including the isospin T [8], one now has to solve the secular equation

$$\sum_{ab} [\langle \acute{a}\acute{b}^{-1}|H|ab^{-1}\rangle_{JMTT_z} - E_n \delta_{\acute{a}\acute{b}^{-1},ab^{-1}}]\, \chi^{JT}_{ab^{-1}} = 0. \tag{2}$$

The matrix element of the Hamiltonian is given by [9]

$$\langle \acute{a}\acute{b}^{-1}|H|ab^{-1}\rangle_{JMTT_z} = (e_{\acute{a}} - e_{\acute{b}})\, \delta_{a\acute{a},b\acute{b}} + \langle \acute{a}\acute{b}^{-1}|V|ab^{-1}\rangle_{JMTT_z}, \tag{3}$$

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† Permenant address: Department of Physics, College of Science, Al-Nahrain University, Baghdad, IRAQ. Email:fouad@if.ufrj.br

where $e_{\acute{a}}$-$e_{\acute{b}}$ is the unperturbed energy of the particle-hole pair obtained from energies in nuclei with A±1 particles.

The matrix element of the residual interaction V is given by the MSDI with the strength parameters A_0=0.8 MeV, A_1=1.0 MeV, B=0.7 MeV and C=−0.3 MeV [9].

$$\langle \acute{a}\acute{b}^{-1}|V|ab^{-1}\rangle_{JMTT_z} = -\sum_{\acute{J}\acute{T}}(2\acute{J}+1)(2\acute{T}+1) \times \begin{Bmatrix} j_{\acute{a}} & j_b & \acute{J} \\ j_a & j_{\acute{b}} & J \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & \acute{T} \end{Bmatrix} \langle \acute{a}\, b|V|a\acute{b}\rangle_{\acute{J}\acute{T}}. \quad (4)$$

The matrix elements of the multipole operators T_J are given in terms of the single particle matrix elements by [7]

$$\langle \Psi_J \| T_{Jt_z} \| \Psi_0 \rangle = \sum_{ab} \chi^{Jt_z}_{ab^{-1}} \langle a \| T_{Jt_z} \| b \rangle\,, \quad (5)$$

where t_z=1/2 for protons and -1/2 for neutrons. The amplitudes $\chi^{Jt_z}_{ab^{-1}}$ can be written in terms of the amplitudes $\chi^{JT}_{ab^{-1}}$ in isospin space as [9]

$$\chi^{Jt_z}_{ab^{-1}} = (-1)^{T_f-T_i}\left[\begin{pmatrix} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{pmatrix}\sqrt{2}\,\frac{\chi^{JT=0}_{ab^{-1}}}{2} + 2t_z \begin{pmatrix} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{pmatrix}\sqrt{6}\,\frac{\chi^{JT=1}_{ab^{-1}}}{2}\right], \quad (6)$$

where

$$T_z = \frac{Z-N}{2} \quad (7)$$

The single particle matrix elements of the electron scattering operator T^η_J are those of Ref.[10] with η selects the longitudinal (L), transverse electric ($E\ell$) and transverse magnetic (M) operators, respectively. Electron scattering form factors involving angular momentum transfer J is given by [10]

$$|F^\eta_J(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)}\,|\langle \Psi_{J_f} \| T^\eta_{Jt_z} \| \Psi_{J_i}\rangle \times |F_{c.m}(q)|^2\;|F_{f.s}(q)|^2 \quad (8)$$

where J_i= 0 and J_f=J for closed shell nuclei and q is the momentum transfer. The last two terms in Eq. (8) are the correction factors for the ($c.m.$) and the finite nucleon size ($f.s.$)[10]. The total inelastic electron scattering form factor is defined as [8]

$$|F_J(q,\theta)|^2 = |F^L_J(q)|^2 + \left[\frac{1}{2}+\tan^2(\theta/2)\right]|F^{Tr}_J(q)|^2, \quad (9)$$

where $|F^{Tr}_J(q)|^2$ is the transverse electric or transverse magnetic form factors.

III. RESULTS AND DISCUSSION

The unperturbed energies for the single particle-hole states for both positive and negative parity states used in this work are adopted from our previous theoretical work (see Table 1 and 2 from Ref.[6]). Higher configurations are included in the calculations when the ground state is considered as a mixture of the $|(1s_{1/2})^4\,(1p_{3/2})^8\rangle$ and $|(2s_{1/2})^4\,(2p_{3/2})^8\rangle$ configurations, such that the ground state wavefunction becomes

$$|\Psi_{00}\rangle = \gamma|\Psi_{00}(1s_{1/2})^4(1p_{3/2})^8\rangle + \delta|\Psi_{00}(2s_{1/2})^4(2p_{3/2})^8\rangle \quad (10)$$

with γ^2+δ^2=1, $\chi^{JT}_{ab_1^{-1}}$=$\gamma\chi^{JT}_{ab^{-1}}$ and $\chi^{JT}_{ab_2^{-1}}$=$\delta\chi^{JT}_{ab^{-1}}$

The excited states is also assumed as a mixture of the particle-hole configurations, $|a_1\, b_1^{-1}\rangle$, $|a_2\, b_2^{-1}\rangle$, $|a_2\, b_1^{-1}\rangle$ and $|a_1\, b_2^{-1}\rangle$, where $|a_1\rangle$=$|a\rangle$=$|n_a\,\ell_a\,j_a\rangle$, $|a_2\rangle$=$|a\rangle$=$|n_a{+}1\,\ell_a\,j_a\rangle$, $|b_1\rangle$=$|b\rangle$=$|n_b\,\ell_b\,j_b\rangle$ and $|b_2\rangle$=$|b\rangle$=$|n_b{+}1\,\ell_b\,j_b\rangle$.

The matrix element given in Eq. (5) is called the model space matrix element, where a and b are defined by the amplitudes given in Tables I and II for the negative and positive parity states, respectively.

The extended space matrix element becomes

$$\langle \Psi_J \| T_{Jt_z} \| \Psi_0 \rangle = \sum_{a_1b_1} \chi^{Jt_z}_{a_1b_1^{-1}} \langle a_1 \| T_{Jt_z} \| b_1 \rangle + \sum_{a_1b_2} \chi^{Jt_z}_{a_1b_2^{-1}} \langle a_1 \| T_{Jt_z} \| b_2 \rangle + \sum_{a_2b_1} \chi^{Jt_z}_{a_2b_1^{-1}} \langle a_2 \| T_{Jt_z} \| b_1 \rangle + \sum_{a_2b_2} \chi^{Jt_z}_{a_2b_2^{-1}} \langle a_2 \| T_{Jt_z} \| b_2 \rangle\,, \quad (11)$$

where

$$\chi^{Jt_z}_{a_1b_1^{-1}} = C_1\,\chi^{Jt_z}_{ab^{-1}}, \quad \chi^{Jt_z}_{a_1b_2^{-1}} = C_2\,\chi^{Jt_z}_{ab^{-1}}, \quad \chi^{Jt_z}_{a_2b_1^{-1}} = C_3\,\chi^{Jt_z}_{ab^{-1}}, \quad \chi^{Jt_z}_{a_2b_2^{-1}} = C_4\,\chi^{Jt_z}_{ab^{-1}}, \quad (12)$$

The values of the parameters C's are given in Table III. The states 1^-, $2_1{}^-$, $2_2{}^-$, 3^- and 3^+ are found experimentally at 18.12 MeV, 19.50 MeV, 22.70 MeV, 18.60 MeV and 20.60 MeV respectively [11]. We obtain the values 18.44 MeV, 19.88 MeV, 23.50 MeV, 18.87 MeV and 27.10 MeV for the states 1^-, $2_1{}^-$, $2_2{}^-$, 3^- and 3^+, respectively.

The 1^- (18.12 MeV), C1+E1 form factor is shown in Fig. 1. The amplitudes χ's reduced by a factor 1.3, to agree with the low q data [7]. This state is dominated by $(2s_{1/2})\,(1s_{1/2})^{-1}$ particle-hole configuration, as given in Table I. The single-particle matrix elements are calculated with the harmonic oscillator wavefunctions (HO) with oscillator parameter $b = 1.64$ fm to agree with the elastic form factor determination [13]. Our results are consistent with the previous calculation of Donnelly [12]

TABLE I: Energies and amplitudes χ^{JT} for J^- $T=1$ state.

Particle-hole configuration $\vert a\, b^{-1}\rangle$	E(1^-) 18.44 MeV χ^{11}	E($2_1{}^-$) 19.88 MeV χ^{21}	E($2_2{}^-$) 23.50 MeV χ^{31}	E(3^-) 18.87 MeV χ^{31}
$(1p_{1/2})\,(1p_{3/2})^{-1}$	0.0473	0.0000	0.0000	0.0000
$(1d_{5/2})\,(1s_{1/2})^{-1}$	-0.1810	0.8314	0.0703	0.9993
$(2s_{1/2})\,(1s_{1/2})^{-1}$	0.9739	0.5430	0.0834	0.0000
$(1d_{1/2})\,(1s_{1/2})^{-1}$	0.1333	-0.1054	0.9936	0.0318
$(1f_{7/2})\,(1p_{3/2})^{-1}$	0.0000	0.0442	0.0000	0.0165
$(2p_{3/2})\,(1p_{3/2})^{-1}$	0.0008	0.0000	0.0222	0.0000
$(1f_{5/2})\,(1p_{3/2})^{-1}$	0.0000	-0.0636	0.0147	-0.0030
$(2p_{1/2})\,(1p_{3/2})^{-1}$	0.0000	0.0000	0.0000	0.0000

TABLE II: Energies and amplitudes χ^{JT} for J^+ $T=1$ states.

Particle-hole configuration $\vert a\, b^{-1}\rangle$	E(3^+)=27.10 MeV χ^{31}
$(1p_{1/2})\,(1p_{3/2})^{-1}$	0.0000
$(1d_{5/2})\,(1s_{1/2})^{-1}$	-0.0475
$(2s_{1/2})\,(1s_{1/2})^{-1}$	0.0000
$(1d_{1/2})\,(1s_{1/2})^{-1}$	0.0000
$(1f_{7/2})\,(1p_{3/2})^{-1}$	0.9461
$(2p_{3/2})\,(1p_{3/2})^{-1}$	-0.3201
$(1f_{5/2})\,(1p_{3/2})^{-1}$	-0.0020
$(2p_{1/2})\,(1p_{3/2})^{-1}$	0.0000

and slightly in better agreement with the experimental data for the momentum transfer region $q \leq 1.0$ fm^{-1}.

The transverse magnetic form factor M2 for the excitation to the $2_1{}^-$, 19.50 MeV state is shown in Fig. 2. The amplitudes have to be enhanced by a factor 1.2 to account for the experimental data. The calculations incorporate the single-particle wavefunctions of the (HO) potential with $b = 1.64$ fm and a value of γ=0.95, to account for the ground state correlation. The data are very well explained for the momentum-transfer $q \leq 3.0$ fm^{-1}.

Figure 3, shows the transverse magnetic form factor

TABLE III: Values of the parameters $C^{,s}$ used in the extended space calculations.

J^π	C_1	C_2	C_3	C_4
3^+	0.92	-0.27	-0.27	0.078
2_1^-	-0.92	0.27	-0.27	0.078
2_2^-	-0.92	0.27	-0.27	0.078

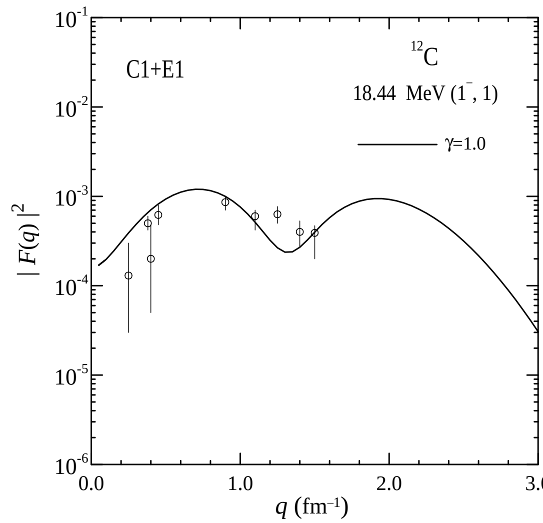

FIG. 1: Form factor for the C1+E1 transition to the (1^-, 1) 18.44 MeV state compared with the experimental data taken from Ref. [12].

M2 for the excitation to the $2_1{}^-$, 22.70 MeV state. The amplitudes have to be reduced by a factor 1.82 to fit the low-q data. The single-particle wavefunctions are those of the (HO) potential with size parameter $b = 1.50$ fm and a value of γ=0.97, to account for the ground state correlation. The experimental data are very well described throughout the momentum-transfer regions and the results are consistent with that of Hicks *et al.*, [14].

The 3^- (18.60 MeV), is dominated by $(1d_{5/2})\,(1s_{1/2})^{-1}$ particle-hole configuration, as given in Table I. The only multipole that contributes to the scattering is the longitudinal C3 multipole as shown in Fig. 4. The calculations incorporate the single-particle wavefunctions of the (HO) potential with $b = 1.64$ fm and γ takes the value 1.0 . The experimental data are very well explained for the momentum-transfer values $q \leq 3.0$ fm^{-1} and the results are consistent with that of Hicks *et al.*, [14] and Yamaguchi *et al.*, [15], where the form factor seems to be a pure longitudinal form factor.

Figure 5, shows the transverse magnetic form factor for the excitation to the 3^+, 20.60 MeV state. The dominated configuration is the $(1f_{7/2})\,(1p_{3/2})^{-1}$ particle-hole configuration, as given in Table II. The only multipole that contributes to the scattering is the magnetic M3 multipole. The amplitudes have to be reduced by factor of 5 to account for the experimental data. The calculations incorporate the single-particle wavefunctions of the (HO) potential with $b = 1.64$ fm, and a value γ=0.7, to account for the ground state correlation. The data are very well explained throughout the momentum-transfer values $q \leq 3.0$ fm^{-1}.

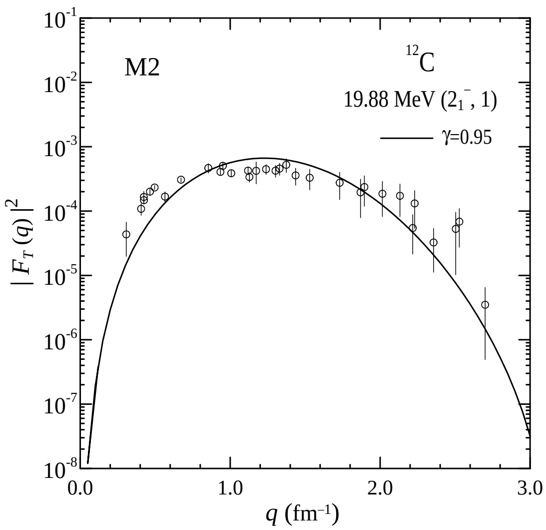

FIG. 2: Transverse magnetic form factor for the M2 transition to the ($2_1{}^-$, 1) 19.88 MeV state compared with the experimental data taken from Ref. [14].

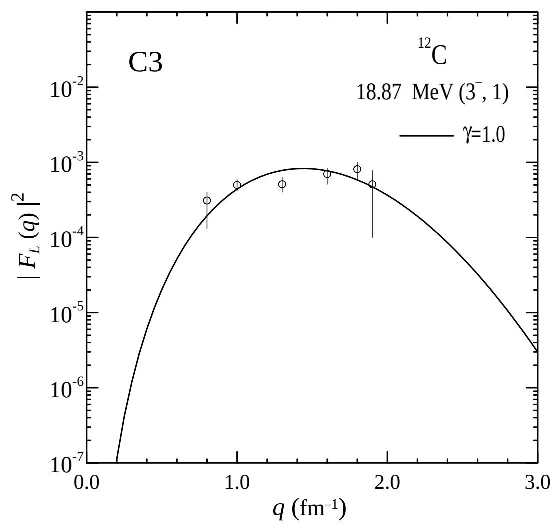

FIG. 4: Longitudinal form factor for the C3 transition to the (3^-, 1) 18.87 MeV state compared with the experimental data taken from Ref. [15].

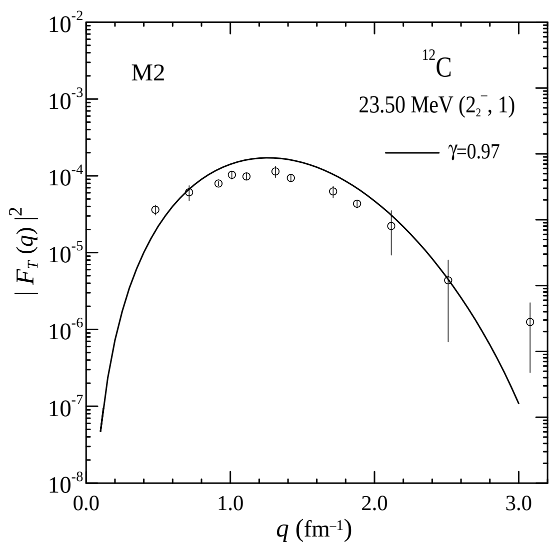

FIG. 3: Transverse magnetic form factor for the M2 transition to the ($2_2{}^-$, 1) 23.50 MeV state compared with the experimental data taken from Ref. [11].

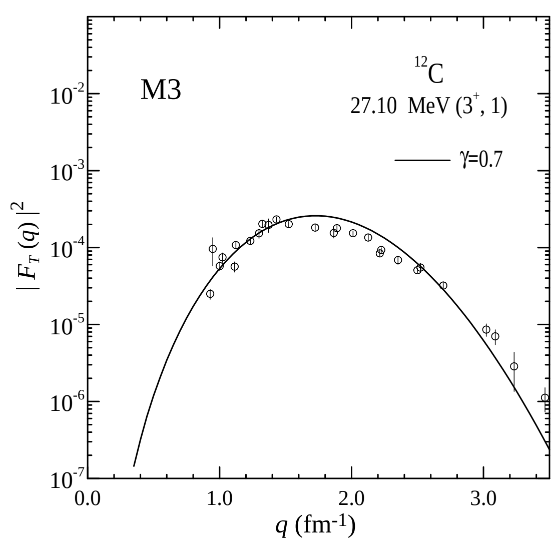

FIG. 5: Transverse magnetic form factor for the M3 transition to the (3^+, 1) 27.10 MeV state compared with the experimental data taken from Ref. [14].

IV. CONCLUSIONS

The inclusion of higher energy configurations in the particle-hole shell model calculation succeeded in describing the form factors for the negative and positive parity states. The amplitudes of the transitions to the negative-parity states considered in this work have to be reduced by a factor 1.3 and 1.82 for the states 1^- and $2_2{}^-$ while the amplitudes for the $2_1{}^-$ state need to be enhanced by factor of 1.2, to describe the low-q data. The amplitudes for 3^+ need to be reduced by a factor of 5. This reduction may be attributed to higher order effects, such as 2p-2h excitations, or even more. Correlation in the ground state wavefunction by mixing more than one configuration are necessary to describe the data. The single-particle wavefunctions of the (HO) potential with size parameter $b = 1.64$ fm chosen to reproduce the root mean square charge radius are adequate to describe the data, except for M2 (23.50 MeV) transition where the b value has to be reduced by a factor 14%.

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